Calculation of Transfer Values — Proposal for STV-PR Rules for Local Government Elections in Scotland

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1 Introduction

The Local Governance (Scotland) Bill [1] will make provision for future local government elections in Scotland to be by the Single Transferable Vote. Those responsible for drafting the legislation have indicated that they do not intend simply to copy the legislation used for the comparable STV elections in Northern Ireland. They believe they can express some points in the counting procedure more clearly. Thus we have a "painless" opportunity to consider some other changes that might usefully be incorporated at the same time. I suggest one of these should be the calculation of transfer values.

2 Precision of calculation

Some discussion in the Election Methods web group [2] prompted me to look in some depth at the calculation of transfer values in STV-PR. The discussion was started by a reference to Wichmann's review [3] of the ERS97 Rules [4]. Wichmann made a number of points about transfer values, starting with what I would call "apparent precision", but going into the arithmetical realities of the truncated calculations adopted in ERS97 and other sets of rules based on Newland and Britton 1972 [5], including those currently used in Northern Ireland. Wichmann's proposal to give results with an actual accuracy of 0.01 votes was to compute transfer values to [(number of digits in total votes) + 1].

Another member of the EM web group drew attention to the procedures of the Australian Electoral Commission [6]. The AEC calculates transfer values to eight decimal places and then truncates as shown in the example on their website. This requirement to calculate to eight decimal places is not specified in any Australian legislation, but only in the AEC's internal working documents [7]. The relevant law [8] makes no reference to the accuracy or precision for any of the STV calculations. The AEC adopted eight decimal places because that was the limit of the desktop calculators available at the time they framed that working rule [7].

The AEC example shows that while they calculate the transfer value of a ballot paper to eight decimal places (8dp) and then use that 8dp result to calculate the transfer values of the votes being transferred, they truncate the candidates' transferred votes to integer values. They do not show decimal parts of a vote anywhere on their result sheets. This truncation to integer values might seem perverse, but does not result in the loss of significant numbers of votes.

In the AEC example there is a surplus of 992,137 votes carried on 1,518,178 papers, of which one candidate receives 1,513,870 papers. The AEC calculation shows an 8dp truncated transfer value of 0.65350505 for each paper. This results in a candidate integer truncated transfer vote of 989,321. The "full" calculation with the 8dp transfer value would have been 989321.69, so they have lost only 0.69 of a vote by integer truncation. This amounts to only 0.000131% of the quota. Had the transfer value been calculated to 15dp (limit of numerical precision for Microsoft Excel 2002), the loss by integer truncation of the votes transferred would have been only 0.700215653, amounting to 0.000133% of the quota.

In contrast, using the ERS/NI rules and calculating the same example to only two decimal places and then truncating, gives a transfer value of 0.65, and a candidate transfer vote of 984,015.50. In this case there would be a loss of 5,306.20 votes from the "true" transfer value, amounting to 1.01% of the quota.

3 Examples from elections

For practical examples I have looked at the immediately available results from the Australian Federal Senate elections in 1998 [9] and the Northern Ireland Assembly elections of 1998 [10]. To make sure there were no complications in the calculations, I looked only at separate transfers arising from the surpluses of candidates whose first preference votes exceeded the quota, i.e. who were elected at stage 1. The relevant figures are in the Tables 1 and 2. In the Australian results they show "non-transferable votes" separately for "exhausted ballots" and for "lost by fraction", ie due to truncation.

The losses arising from truncation are expressed as percentages of the quotas for the relevant elections because this offers the most valid basis for comparisons among the different elections. The results are sorted in ascending order by the size of these percentages. The losses in the Australian transfers range from 0.0043% to 0.032%. In only six of those 14 transfers did the loss exceed 0.01% of the quota. The losses in the Northern Ireland transfers range from 0.10% to 1.36%. In five of those 23 transfers the loss exceeded 1.0% of the quota.

The size of the loss in any individual transfer will depend on just how the calculation tumbles out as that will determine the size of the fraction truncated. For example, in the Newry and Armagh election the transfer value was 0.43 (excluding 222 exhausted papers), leading to a loss of 0.0077245 votes on every one of the 13,360 papers actually transferred. In the Australian elections the losses are increased by the large numbers of candidates who stand and to whom transfers are made.

4 Proposal for change

It now seems clear to me that when the STV rules were formalised for Newland and Britton and the Northern Ireland STV regulations in 1972, there was a confusion of two objectives. It is illogical to calculate transfer values to only two decimal places if candidates' votes are to be recorded to of 0.01 of a vote. This approach was probably taken because the 'Senatorial Rules' [11], devised to remove the element of chance when selecting full value ballot papers for the transfer of surpluses, had given each valid ballot paper a value of one hundred before any calculations were done.

For public elections, with large numbers of electors, there is no intrinsic merit in recording candidates' votes with a precision greater than one vote, provided that does not result in the loss of significant numbers of votes. For elections with small numbers of electors (quota less than 100), there may be a benefit in recording candidates' votes with greater precision, perhaps to 0.01 of a vote. Whatever level of precision is required in the recorded vote, calculating transfer values of ballot papers to only two places of decimals is not consistent with that reported precision. There may be a theoretical case for varying the numbers of decimal places in the calculation according the magnitude of the numbers of votes, but the practical approach of the AEC has been shown to give very satisfactory results.

The AEC adopted eight decimal places for the calculation of transfer values because that was the capacity of the desktop calculators available at the time. Most currently available electronic calculators (hand-held and desktop models) display eight decimal digits, i.e. it is possible to enter '12345678' but not '123456789'. However, when a division to obtain a transfer value is made on such a calculator, the result does not contain eight decimal places, but only seven. Thus, to use the example from the AEC website, (surplus = 992137; transferable papers = 1518178), an 8-digit electronic calculator would display a result of 0.6535050 and not the 0.65350505 quoted. It would be possible to obtain eight significant figures on such a calculator by scaling the calculation, eg 992137 / 151817.8 or 9921370 / 1518178. The transfer value would then be displayed as '6.5350505'. However, there would an additional risk of mistakes being made if calculations were scaled in this way and the increase in precision would be very small.

Taking a practical approach, I would recommend that transfer values should be calculated to 7 decimal places, reflecting the capacity of the commonly available electronic calculators. If the calculation loss is minimised in this way, there is then no need to record decimal fractions of votes for each candidate on the result sheet. The loss that would be incurred in discarding the fractional values when summing the votes for each candidate is very small compared to the calculation loss. This would greatly simplify the presentation of STV-PR result sheets for public elections.

Table 1 Australian Federal Senate Elections 1998
Non-transferable Votes arising on Transfer of Surpluses from First Preferences of Candidates elected at Stage 1

State	Total Vote	Quota	Candidate	Candidate's F P Vote	Surplus	Candidates receiving votes	Exhausted Ballots	Lost by Fraction	LbF as Percentage of Quota
NSW 2	3,755,725	536,533	Heffernan	1,371,578	835,045	35	12	23	0.0043%
NSW 1	3,755,725	536,533	Hutchins	1,446,231	909,698	39	18	25	0.0047%
QLD 3	2,003,710	286,245	Hill	295,903	9,658	15	1	14	0.0049%
VIC 2	2,843,218	406,175	Troeth	1,073,551	667,376	27	9	22	0.0054%
VIC 1	2,843,218	406,175	Conroy	1,148,985	742,810	28	10	24	0.0059%
QLD 1	2,003,710	286,245	McLucas	653,183	366,938	31	15	23	0.0080%
QLD 2	2,003,710	286,245	Parer	568,406	282,161	26	8	24	0.0084%
SA 2	946,816	135,260	Bolkus	301,618	166,358	23	6	13	0.0096%
WA 1	1,063,811	151,974	Ellison	405,617	253,643	26	10	16	0.0105%
WA 2	1,063,811	151,974	Cook	366,874	214,900	33	11	16	0.0105%
SA 1	946,816	135,260	Vanstone	381,361	246,101	27	8	17	0.0126%
ACT	197,035	65,679	Lundy	83,090	17,411	15	4	10	0.0152%
TAS 2	308,377	44,054	Abetz	98,178	54,124	18	18	12	0.0272%
TAS 1	308,377	44,054	O'Brien	121,931	77,877	22	30	14	0.0318%

Table 2 Northern Ireland Assembly Elections 1998
Non-transferable Votes arising on Transfer of Surpluses from First Preferences of Candidates elected at Stage 1

State	Total Vote	Quota	Candidate	Candidate's F P Vote	Surplus	Candidates receiving votes	Non-transferable votes	NTV as Percentage of Quota
East Antrim 2	35,610	5,088	Neeson	5,247	159	11	4.89	0.10%
Belfast East 1	39,593	5,657	Robinson	11,219	5,562	15	6.00	0.11%
South Antrim	43,991	6,285	Wilson	6,691	406	9	10.96	0.17%
Belfast North 2	41,125	5,876	Maginness	6,196	320	15	12.25	0.21%
Upper Bann 1	50,399	7,200	Trimble	12,338	5,138	16	20.30	0.28%
Belfast West 1	41,794	5,971	Adams	9,078	3,107	13	22.10	0.37%
North Antrim	49,697	7,100	Paisley	10,590	3,490	15	28.30	0.40%
East Londonderry	39,564	5,653	Campbell	6,099	446	10	25.44	0.45%
West Tyrone	45,951	6,565	Gibson	8,015	1,450	12	32.29	0.49%
Mid-Ulster 2	49,798	7,115	McGuinness	8,703	1,588	7	45.40	0.64%
Fermanagh & South Tyrone	51,043	7,292	Gallagher	8,135	843	11	50.80	0.70%
Mid-Ulster 1	49,798	7,115	McCrea	10,339	3,224	10	49.60	0.70%
Upper Bann 2	50,399	7,200	Rodgers	9,260	2,060	14	55.36	0.77%
Belfast North 1	41,125	5,876	Dodds	7,476	1,600	15	45.79	0.78%
Belfast West 2	41,794	5,971	Hendron	6,140	169	10	50.80	0.85%
North Down	37,313	5,331	McCartney	8,188	2,857	18	47.55	0.89%
Strangford 1	42,922	6,132	Robinson	9,479	3,347	18	59.80	0.98%
East Antrim 1	35,610	5,088	Beggs	5,764	676	14	49.99	0.98%
Foyle	48,794	6,971	Hume	12,581	5,610	14	69.60	1.00%
Belfast East 2	39,593	5,657	Alderdice	6,144	487	18	58.81	1.04%
Strangford 2	42,922	6,132	Taylor	9,203	3,071	20	73.61	1.20%
South Down	51,353	7,337	McGrady	10,373	3,036	16	90.76	1.24%
Newry & Armagh	54,136	7,734	Mallon	13,582	5,848	13	104.92	1.36%

5 Benefits in local government elections in Scotland

The numbers of electors in the constituencies in both the Australian Federal Senate elections and the Northern Ireland Assembly elections are considerably larger than those likely in the multi-member wards for local government elections in Scotland. It is, therefore, useful to make an assessment of the potential effects of changing the precision of calculation of transfer values from 2dp to 7dp using local data.

For this example I have used Glasgow City Council which has an electorate of 453,552 and 79 councillors. I have examined two possible implementations of STV-PR: nine 8-member wards plus one 7-member ward; and nineteen 4-member wards plus one 3-member ward (Table 3). I have assumed there would be equal numbers of electors per councillor in all wards and a turnout of 50%. I have also assumed that the Labour Party would get 47.58% of the first preference votes (= city-wide average in the 2003 FPTP council elections), that 75% of those first preference votes would be for the party's leading candidate in the ward and that all those papers would be transferable. For the calculation with 7dp I have also truncated the transferred votes to integer values as I recommend above. The results in Table 3 show that the effect of truncating the calculation of transfer values at 2dp could be considerable even in the smaller 4-member wards. The losses when the calculation is truncated at 7dp are negligible.

Table 3 Comparison of Effects of Calculating Transfer Values to 2dp and 7dp

Implementation	*8-member ward*	*4-member ward*
Electorate	45,929	22,964
Valid votes	22,964	11,482
Quota	2,552	2,297
Party FP votes	10,926	5,463
Leading candidate's FP votes	8,194	4,097
Surplus for transfer	5,642	1,800
Transfer value 2dp	0.68	0.43
Transferred votes 2dp	5,571.92	1,761.71
Votes lost by truncation at 2dp	70.08	38.29
Votes lost as percentage of quota	2.75%	1.67%
Transfer value 7dp	0.6885525	0.4393458
Transferred votes 7dp	5641	1,799
Votes lost by truncation at 7dp	1	1
Votes lost as percentage of quota	0.039%	0.044%

The actual loss in transfer value due to truncating the calculation at 2dp compared to truncating at 7dp can vary from 0.0000000 to 0.0099999. The general effect can be assessed by considering only the loss that occurs in the third decimal place. The results in Table 4 have been calculated using the same two example wards as above. The ten potential losses all have equal probabilities of occurrence. The loss due to truncation at 2dp in the 8-member ward will exceed 1% of the quota in six cases out of ten and will exceed 2% in three cases out of ten. Even in the smaller ward, the loss due to this truncation will exceed 1% of the quota in four cases out of ten. These losses are substantial and could be avoided by a simple change to the rules for STV-PR elections.

Table 4 Loss of Votes due to Truncation of Transfer Value before 3dp

Implementation	*8-member ward*		*4-member ward*	
Transferable papers	8,194		4,097	
Loss in transfer value	Votes lost	% of quota	Votes lost	% of quota
0.000	0	0.00%	0	0.00%
0.001	8	0.31%	4	0.17%
0.002	16	0.63%	8	0.35%
0.003	24	0.94%	12	0.52%
0.004	32	1.25%	16	0.70%
0.005	40	1.57%	20	0.87%
0.006	49	1.92%	24	1.04%
0.007	57	2.23%	28	1.22%
0.008	65	2.55%	32	1.39%
0.009	73	2.86%	36	1.57%

6 References

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