Meek *versus* Warren

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1 Where we agree (I.D. Hill and C.H.E. Warren)

We admire traditional STV methods (Newland and Britton rules [1] and other similar methods) as being a good approximation to what STV is trying to achieve, while being easy enough to do by hand within a reasonable length of time, but in this electronic age, we ought to do better than that. Of course we accept that the ability to count by hand is an advantage; but does such an advantage justify the consequence that, quite often, the set of candidates who best meet the voters' wishes are not elected? We think not. But if we seek to campaign for something better, we need to agree on the better thing that we should support.

We agree that fairness is of prime concern in a voting system, but it is a tricky concept — one only has to listen to politicians all claiming that taxation, for example, must be fair ("and must be seen to be fair" as if that addition helped), while totally disagreeing with each other about what is fair and what is not.

The Meek method [2] and the Warren method [3] are very similar to each other but, in deciding how much of each vote is retained by an elected candidate and how much is passed on to the next choice, the Meek method uses multiplicative 'keep values' but the Warren method uses additive 'portions apportioned'. We here denote the Meek keep value and the Warren portion apportioned for candidate C as c_m and c_w respectively. These quantities have a value between 0 and 1, and they are calculated so that, if a candidate has a surplus, their use reduces the vote for that candidate to just the quota. The calculation of these quantities so that they meet this requirement is a mathematical problem, usually requiring a computer. All that we need to know in this paper is that they can be calculated.

With the Meek method c_m is defined as the proportion of the vote that is passed to candidate C which candidate C retains, so that $(1 - c_m)$ is the proportion of that vote that is passed on. In the case of a ballot that reads ABC...

the portion of vote which A retains is a_m
the portion of vote which A passes on to B is
$(1 - a_m)$
the portion of vote which B retains is
$(1 - a_m)b_m$
the portion of vote which B passes on to C is
$(1 - a_m)(1 - b_m)$
the portion of vote which C retains is
$(1 - a_m)(1 - b_m)c_m$
the portion of vote which C passes on is
$(1 - a_m)(1 - b_m)(1 - c_m)$
and so on.

From the above statements we see why the Meek keep values are called multiplicative.

With the Warren method c_w is defined as the portion of a vote that is apportioned to candidate C if such apportionment is possible. In the case of a ballot that reads ABC...

the portion of vote which is apportioned to A is a_w
if $a_w + b_w > 1$, the portion of vote which is
apportioned to B is $(1 - a_w)$
and nothing is apportioned to C and beyond
if $a_w + b_w \leq 1$, the portion of vote which is
apportioned to B is b_w
if $a_w + b_w \leq 1$ and $a_w + b_w + c_w > 1$,
the portion of vote which is
apportioned to C is $(1 - a_w - b_w)$
and nothing is apportioned beyond
if $a_w + b_w + c_w \leq 1$, the portion of vote which is
apportioned to C is c_w
and so on.

From the above statements we see why the Warren portions apportioned are called additive.

Although a Meek keep value c_m may, in some circumstances, turn out to have the same value as a Warren portion apportioned c_w, in general their numerical values are different.

The methods are equally easy to program for a computer and, for real voting patterns as distinct from test cases, they nearly always produce the same answers, not in numerical terms but in terms of which candidates are elected and which are not. In those circumstances, we agree that it does not matter too much which is used, so it is preferable to support the one that is better in principle — but which one is that?

We recognise that impossibility theorems, such as Woodall's theorem [4], show that to seek an absolute ideal is a 'wild-goose chase'. It follows that it will always be possible to produce particular examples that tell against any given method. Unlike proving a proposition in pure mathematics, where one counter-example is enough to demonstrate that we have failed, here we always need to look at examples in a comparative sense, not an absolute sense, deciding which faults to allow for the sake of avoiding others.

2 Why I prefer the Meek method (I.D. Hill)

To my mind the essence of STV is this — if we have a quota of 7, and 12 identical votes putting A as first preference and B as second (with no others for A) then 7 votes must be held for A as a quota while the other 5 are passed to B and, from that point on, behave exactly as if they had originally been 5 votes for B as first preference. The fact that those voters had A as first preference, and A has been elected, has been fully allowed for in holding 7 votes back and the other 5 votes are now simply B votes.

In practice, we never get such identical votes, so the only fair way of doing things is, instead of holding 7 complete votes back and passing on 5 complete votes, to hold back $\frac{7}{12}$ of each vote and pass on $\frac{5}{12}$ of each vote, but the principle, that the 12 votes each of value $\frac{5}{12}$ should together have the same power as 5 complete votes, remains the same. This principle is fulfilled by the Meek method, but not by the Warren method. Because perfection is impossible, it could be that some advantage could be shown by the Warren method that would outweigh this disadvantage, but I am not aware that any advantage has been claimed for it that is strong enough to do so.

If, at the next stage, we have 5 votes with B as first preference, plus our 12 votes each now of value $\frac{5}{12}$, we have 10 votes altogether pointing at B. Only 7 are needed for a quota so $\frac{7}{10}$ needs to be retained allowing $\frac{3}{10}$ to be passed on, so the 5 votes are passed on with a value of $\frac{3}{10}$, giving them a total power of $1\frac{1}{2}$ votes. If the 12 votes are passed on with a value of $\frac{5}{12}$ times $\frac{3}{10}$, that gives them a total power of $1\frac{1}{2}$ votes too, showing that 12 each of value $\frac{5}{12}$ are being treated just like 5. To get that effect necessarily requires a multiplicative rule, not an additive rule.

To look at it from a slightly different angle, the rule should be that the proportions of the total vote for a candidate that come from different sources, and are used in deciding that the candidate can now be elected, should be maintained in the amounts of vote retained and transferred. Thus, in the same example, the votes from the AB voters and from the B voters that are used to decide to elect B are in proportion 1 to 1, whether the Meek or the Warren method is used. With Meek, the votes retained from the two groups are $3\frac{1}{2}$ and $3\frac{1}{2}$, also 1 to 1, and those transferred are $1\frac{1}{2}$ and $1\frac{1}{2}$, also 1 to 1. With Warren, the votes retained are $4\frac{16}{17}$ and $2\frac{1}{17}$, or 2.4 to 1, and those transferred are $\frac{1}{17}$ and $2\frac{16}{17}$, or 1 to 50, devoid of all the proportionality that I believe they should have.

The Meek method is able to promise voters that once their first n choices have all had their fates settled, either as excluded or as elected with a surplus, a fair share of their vote will be passed to their $(n + 1)$th choice, unless no more transfers are possible because all seats are now filled. How much is a fair share may, perhaps, be arguable (though I do not personally see it as such) but it cannot possibly be zero, which the Warren method often makes it.

Thus the basis of STV in Meek mode is that everything has to be done in proportion to the relevant numbers at the time. This means that if we have 1 ballot paper of value 1 pointing at XY, and n ballot papers each of value $\frac{1}{n}$ pointing at XZ, and X's papers are to be redistributed, then what happens to Y and to Z from those papers should be identical.

Suppose 8 candidates for 7 seats, counted by Newland and Britton rules. If there are 40 votes reading 5 ABCG, 5 ABCH, 5 ABDG, 5 ABDH, 5 ABEG, 5 ABEH, 5 ABFG, 5 ABFH, it is evident from the symmetry that ABCDEF must be elected but the final seat is a tie between G and H. If, however, there is a 41st vote reading BH, that ought to settle it in favour of H, but those rules declare it still to be a tie between G and H to be settled at random. Either Meek or Warren counting would have awarded the seat to H.

However, suppose the 41st vote, instead of being just BH reads BCDEFH. Again Newland and Britton rules fail to discover that the symmetry has been broken, and incorrectly call it a GH tie. But now so do Warren rules. With Meek rules, only 0.012 of the vote gets through as far as H, but that is enough to tilt the balance to get the right result.

In the past, when Hugh Warren and I have argued about this, each of us has, from time to time, put forward an example with an 'obviously right' answer which the other one's preferred method failed to find. However, with those examples, the other one of us never accepted that the answer in question was 'obviously right'. It was therefore necessary to produce something where the answer could not be denied. I claim to have done this with the example: 4 candidates for 3 seats, and just 3 votes: 1 ABC, 1 BC, 1 BD. Without even knowing anything about STV, it must be clear that ABC is a better answer than ABD. Meek does elect ABC, but Warren says that C and D tie for the third seat and a random choice must be made between them. Unless something equally convincing can be found that points the other way, that seems to me to be conclusive.

So far as I am aware, the only actual advantage claimed for Warren over Meek is that it is supposed to give consistency when some voters change the order of two candidates both of whom are elected anyway. This seems to me to be only a very slight advantage, and Warren rules do not always succeed even in that. With 5 candidates for 4 seats and votes 9 ABCD, 8 BD, 8 CE, 7 D, 7 E, either Meek or Warren elect ABCD. But if the ABCD votes had been ACBD instead, either Meek or Warren would elect ABCE.

The difference arises from the fact that one quota of votes is necessarily ineffective and changing the order of some preferences can change which votes those are and thus, in marginal cases, affect the result. I suggest that in practice any such inconsistency would never be noticed and is of very minor importance compared with making the count so that everything is kept in proportion to the numbers concerned.

I am less convinced than I was even that such behaviour can be called an anomaly. If two candidates are both elected anyway, it would seem at first sight that, if some voters change the order of those two, it ought not to affect who else gets elected, but is that really a good rule? In this example, there is some connection between B and D, and between C and E. We do not know what the connection is, but it is clearly there since every voter putting B first puts D second, while every voter putting C first puts E second. The second choice of the A supporters is then saying what they think about the feature that gives the connection. In such circumstances, it does not seem unreasonable that if the A voters prefer B to C that helps D, but if they prefer C to B that helps E, particularly when the first preferences for D and E are tied.

Overall, while accepting that the Warren method works quite well, it does not seem to me to have any real advantage over the Meek method, and its failure to meet what I regard as basic requirements can sometimes lead to a result that I would think unfortunate. Given how wrong it seems, I am surprised that it works as well as it does.

3 Why I prefer the Warren method (C.H.E. Warren)

I prefer the Warren method because I consider it to be based on a better principle.

The main principle behind the Warren method (given as the second principle in [3]) can be stated as: if a voter votes for candidates A, B, C in that order, and if candidates A and B each have a surplus of votes above the quota, then, on principle, no portion of the vote for ABC shall be credited to candidate C unless the voter has contributed, as far as he is able, the same portion of his vote to the election of candidate B as other voters who have contributed to the election of candidate B.

The main principle behind the Meek method (given as principle 2 in [2]) can be stated as: if a voter votes for candidates A, B, C in that order, and if candidates A and B each have a surplus of votes above the quota, then, on principle, a portion of the vote for ABC shall be credited to candidate C.

These different principles lead to the different rules as set out in paragraphs 3 to 8 of section 1.

I think that whether one prefers the Meek method to the Warren method, or vice versa, should be based on principle, and I prefer the principle upon which the Warren method is based. As stated in paragraph 8 of section 1, because of the impossibility theorems, it will always be possible to produce particular examples that tell against any given method. So I prefer to rest my case on the matter of principle, rather than on seeking examples of where the Warren method gives a 'better' result than the Meek method. Nevertheless, an example will be given, not with the object of showing that one method gives a better result than the other, but of showing how the two methods can give different results.

Consider the following election for 3 seats by 39996 voters, for which the quota is 9999.

10000	vote	ABC
100	vote	AE
10000	vote	BD
9998	vote	C
9898	vote	D

The numbers have been chosen so that, unlike the situation in real elections, the count can be done manually.

Under the Meek method the count can be portrayed as follows:

Voter	Number of such voters	Portion of vote contributed by each voter to each candidate A	B	C	D	E
Keep value		0.99	0.99	1	1	1
ABC	10000	0.99	0.0099	0.0001	0	0
AE	100	0.99	0	0	0	0.01
BD	10000	0	0.99	0	0.01	0
C	9998	0	0	1	0	0
D	9898	0	0	0	1	0
Total vote for each candidate		9999	9999	9999	9998	1

Under the Warren method the count can be portrayed as follows:

Voter	Number of such voters	Portion of vote contributed by each voter to each candidate A	B	C	D	E
Portion apportioned		0.99	0.9899	1	1	1
ABC	10000	0.99	0.01	0	0	0
AE	100	0.99	0	0	0	0.01
BD	10000	0	0.9899	0	0.0101	0
C	9998	0	0	1	0	0
D	9898	0	0	0	1	0
Total vote for each candidate		9999	9999	9998	9999	1

We see from these tables that the Meek method elects candidates A, B, C, whereas the Warren method elects candidates A, B, D.

We observe that the Meek and Warren methods are in agreement as to the portion of vote that each of the ABC voters and the AE voters contribute to candidate A, which is in keeping with the Warren principle that all contributors to the election of a candidate should contribute the same portion of their vote.

We observe that the Meek and Warren methods differ in the portion of vote that each of the ABC voters, and each of the BD voters, contribute to candidate B. Both methods ask the BD voters to contribute closely 99% of their vote to candidate B, and ask the ABC voters to contribute only closely 1% to candidate B. The Warren method accepts this difference, because, although it would have preferred that all groups of voters contributed the same portion, it recognises that the ABC voters did use up all that was left of their vote after contributing to candidate A, and could not contribute more.

The Meek method is desirous that, if a voter votes for a candidate who is elected with a surplus, then that voter should not be asked to contribute so much of his vote to that candidate that he has nothing to pass on. Accordingly, although each ABC voter is contributing only closely 1% of his vote to the election of candidate B, compared with the 99% that each BD voter is contributing, Meek's principle requires that the ABC voters shall contribute slightly less than 1% of their vote to the election of candidate B in order that a portion, which amounts to about one ten-thousandth of a vote, shall be passed to candidate C.

This shows what the difference between the Meek and Warren methods amounts to. In my opinion the difference raises the question as to whether the ABC voters, who have contributed only closely 1% of their vote to the election of candidate B, whereas the BD voters have contributed closely 99% towards the same end, merit the right, in these circumstances, to pass on a portion of their vote to candidate C, as Meek's principle requires, at the expense of expecting the BD voters to bear even more of the burden of electing candidate B. If one thinks that the right should be afforded, then one should prefer the Meek method. But if one thinks that it would not be fair to afford this right, then one should prefer the Warren method.

4 References

[1] R.A. Newland and F.S. Britton. *How to conduct an election by the Single Transferable Vote*. 2nd edition. Electoral Reform Society. 1976.

[2] B.L. Meek. A new approach to the Single Transferable Vote. *Voting matters*, issue 1, 1–11. 1994.

[3] C.H.E. Warren. Counting in STV elections. *Voting matters*, issue 1, 12–13. 1994.

[4] D.R. Woodall. An impossibility theorem for electoral systems. *Discrete Mathematics*, 66, 209–211. 1987.